Kirchhoff vs. Competitive Electricity Markets: A Few Examples

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Abstract—**Electric power is often regarded as a homogeneous commodity due to the ubiquity of the transmission grid. This paper, however, presents a collection of cases in which the physical laws governing network flows can have anomalous and unexpected market implications. For example, reactive power requirements can affect optimal unit commitment and impact real power prices in otherwise competitive markets. Network topology and constraint interactions can result in other unwelcome market phenomena, such as large price differentials within a congestion zone, nodal prices well above the highest offer and "cascading market power".**

I. INTRODUCTION

The last decades of the 20th century have witnessed an unprecedented globalization of markets for many goods. This was not a fortuitous event; current economic thought states that efficiency and economic benefits are better sought by means of open market institutions, and many political and economic forces actively promote such markets. Competition in open markets drives suppliers to attain efficiency and the corresponding productivity gains will be reflected in lower prices, to the benefit of consumers. Already many industries such as air travel, gas, and electricity, have undergone radical transformations to insure that an open market structure exists.

Electricity is a unique good. It cannot be stored without substantial infrastructure, such as pumped storage facilities, and thus most of it is produced and delivered instantaneously through extensive networks of power transmission lines collectively known as "the grid". It is fair to say that all power outlets in the country are electrically connected by means of this grid. However, zones have been defined following the traditional territories of the original electric utilities. Centralized within the zone are certain control operations as well as the general operation of the market that determines prices within that zone. Electric power is also traded across zones, so zonal boundaries are closely monitored to verify that the scheduled transfers take place. Within zones, electric power is regarded as a homogenous good and injected to and extracted from the grid accordingly. Any price differentials resulting from congestion are computed internally.

Independent of the economic and jurisdictional structure of the grid, physical laws governing the behavior of electrons in conducting mediums are obeyed at every instant in time, regardless of whether or not the results are convenient for the appropriate delivery of electricity. Indeed, Kirchhoff's laws, which govern electric flow from a macroscopic perspective, are met at every space and time scale of interest. Voltages, phase angles, and active and reactive injections must at all times obey Kirchhoff's laws. Hence these variables always constitute a solution to a set of algebraic constraints which are functions of the parameters of the transmission lines in the grid. In fact, so-called smart markets work by solving a constrained resource scheduling problem in which the production to be met is the forecasted demand, but the solution is subject to constraints imposed by Kirchhoff's algebraic constraints and other operational constraints such as voltage limits, line thermal transfer limits, the generators' capacities, and other limits which have their origin in security considerations. The offers submitted to the market by the operators of generation are used as production cost information and an optimization algorithm is used to solve the scheduling problem. The Lagrange multipliers provided by the algorithm are then used to compute nodal prices. The market clearing mechanism, which can be modeled after an auction institution such as *last accepted offer* (LAO) or *first rejected offer* (FRO), clears the market based on these prices.

Electric power markets are not the only markets that exhibit constraints in the minimum-cost scheduling problem. Gas delivery over pipelines is subject to pipeline congestion; airline traffic is subject to the capacity of hubs. The economic theory that provides the basis for the efficient operation of these markets can accommodate linear constraints easily. However, the physical constraints imposed by the grid are in general much harder to model than the (usually) linear constraints found in the markets for other goods. For one thing, many *state variables* in addition to generation quantities must be considered in the optimization problem. Voltages, phase angles, phase

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shifter and transformer tap settings, and switched capacitor banks all enter the problem in one way or another. In addition, the constraints imposed by Kirchhoff's laws are highly nonlinear. Finally, many other constraints of engineering consideration, such as thermal transfer limits, are also complex nonlinear functions of the state variables. The purpose of this paper is to illustrate some anomalous market situations brought about by these unique kinds of constraints. These situations show that electricity market designers cannot blindly apply the market institutions that have worked for other goods to the electric power industry and expect it to work. In particular, the network and all of its complexities must be considered from the initial design. Most of these examples have surfaced in the course of researching other aspects of deregulation in the electric power industry; they were found, not concocted. These unexpected situations are in contrast to the more predictable congestion issues in the markets for other goods.

There is already a large amount of research involving market power and implications of congestion in electric power markets. Rassenti, Smith and Wilson have performed studies employing experimental economics on radial networks [10]; Hogan studied AC power flow-based triangular networks [2, 4]. Bunn [9] attributes the lack of competition and volatility in the UK market to ownership and cost curve anomalities. However, few works deal with the intricacies of accurate network models [8]. Our examples typically involve larger, more realistic networks based on IEEE test models, and in part thanks to that we have discovered a richer set of anomalies than has previously been explored.

When the most accurate model of the grid is used for market clearing, the optimization problem, called an *optimal AC power flow*, is a formidable mathematical program to solve. Thus, in practice, very often a simplified, linear model of the grid is used, and later ad-hoc procedures or balancing mechanisms, either rule-based or market-based, are employed to adjust the dispatch to meet all constraints. This inaccuracy in the model on which the market-clearing is based can increase opportunities for gaming [2], adding an extra layer of inefficiency to the market. To avoid this source of inefficiency so that we can better blame the grid alone if necessary, this paper considers studies in which the market-clearing mechanism is based on a full AC optimal power flow, with all nonlinearities included.

II. REACTIVE MANAGEMENT

While many proposals exist on how to deal with reactive power issues, to date no market has addressed the problem of market-based reactive management comprehensively. Nobody disputes its importance, but it has been hard to arrive at a consensus. The network requires reactive power, simply because it is energized, in order to provide an adequate voltage profile (without which electricity is useless to consumers) and to give extra degrees of freedom to the system operator so that the network can be controlled in an appropriate manner. Having these extra degrees of freedom allows the system operator to configure the system to achieve the best use of the transmission capacity. Total cost, system losses, nodal price differentials across congested lines and operational voltage limits are all assessed in a dispatch produced by an OPF. The higher the number of controllable reactive injections in the network, the more freedom the system operator has to optimize the dispatch. However, the most important sources of reactive power are the generators themselves, which, by virtue of the unit commitment decision, can link their reactive power output to the purchase of at least an initial block of active power from them.

Two examples are shown in this section. The first one illustrates a generator that becomes a must-run unit at high load times because of its ability to provide voltage support. The second illustrates how, under low load conditions, a unit that fits the offer profile of a peaking unit is actually committed because many other units have been decommitted, and in such conditions the ability to exert reactive control by the system operator becomes diminished.

A. VAR-RELATED MUST-RUN UNITS

The system discussed in this example arose during the design of an experiment using the PowerWeb [7] environment for simulation of electric power markets. The underlying topology is that of the IEEE 30-bus system [1]. The system was used to conduct an experiment that tested the performance of an auction institution using LAO or FRO clearing mechanisms; the market was divided in high and low load periods. It turned out that in the high load period, generator 4 at bus 27 (see Figure 2) was a must-run generator because of voltage limit violations at bus 30. The subject in control of that generator quickly learned to offer the initial block of power at the reservation price. The first block from a generator must be accepted wholly or rejected completely because of physical lower generation limit issues. Thus, the initial 5MW of power offered by that generator sold at the reservation price.

B. VAR-RELATED FLEXIBILITY OF DISPATCH

The second example surfaced while testing the unit commitment algorithm described in [5]. This algorithm is based on Lagrangian relaxation but permits the inclusion of nonlinear AC OPF constraints. A realistic 168-hour load profile was used to test this algorithm and the generation was a mixture of base, coal and peaking

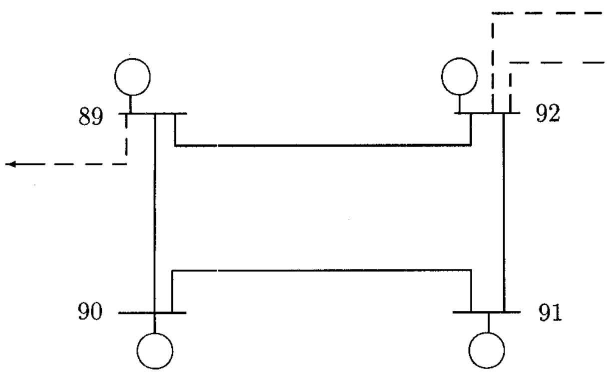

Fig. 1. Section of IEEE 118 bus system

units, the difference being in the pricing scheme, start-up costs and minimal shut-down and start-up times. Peaking units would typically have high generation costs, medium startup costs and short minimum star-up and shut-down times. Thus, it was expected that units with such characteristics would be turned on only during the daytime peaks by the algorithm.

The test case used was the IEEE 118-bus test system. This test case is characterized by a high generator to bus ratio; with 54 generators and 118 buses, the system is very flexible when it comes to reactive dispatch, provided that most generators are online. This, however, was not the case for the commitment schedules chosen by the algorithm for the nightly valley load; many generators were shutdown, and the system no longer offered so much freedom for reactive dispatch. In particular, it was noticed that some of the best solutions found by the unit commitment algorithm had a few peaking units committed at night. These units were dispatched against their lower operating limit, since their power is expensive; however, their reactive capability was used without restraint. In Figure 1, the units at buses 90 and 91 are peaking units, and the unit at bus 89 is a big baseload unit that is always dispatched at its maximum operating limit. The unit at bus 92 was characterized such that it is usually turned on starting with the shoulder load level, but is off-line at night. Furthermore, the path 89-90-91-92 is considerably more lossy than the more direct parallel path 89-92. Apparently, the algorithm turns on the units at buses 90 and 91 to better manage the losses along the 89-90-91-92 path, and to help channel more of the power being injected at bus 89 to the 89-92 line, and then to the rest of the network. This sort of behavior is unexpected and challenges traditional beliefs about optimal unit commitment, and, in a deregulated market setting, can lead to strategic behavior and market power.

III. CASCADING MARKET POWER

In a perfect competition setting with LAO clearing, the price setter can raise the clearing price only as high as the price of the first rejected block, and it is the competition between the first rejected and the last accepted blocks that can promote marginal price offers. When there is congestion, however, generators inside a load pocket can raise the price on the portion of the demand inside the pocket that cannot be served from outside. However, under certain topological configurations it is possible that the ability to set the price "cascades" upstream along the paths of the transmission lines going to the load pocket, and it may be possible for a generator to unilaterally raise the price thanks to this cascade effect. Consider again a modified version of the IEEE 30-bus system as shown in Figure 2. Here, Area 1 is isolated by congestion. Marginal offering on the part of the generators results in the prices, quantities and earnings shown in the boxes next to each generator in Figure 2. Because of congestion, however, generators #1 and #2 only compete with one another for some part of their local demand and through tacit collusion may be able to inflate the price in Area 1. With doubled offer prices, their earnings are increased substantially, as can be seen in Figure 3. Interestingly, however, in this situation, generator #4, which is outside the load pocket but on one of the major paths going into Area 1, can unilaterally increase its earnings by raising its offer, as shown in Figure 4. In this case, outside the load pocket, the network has reduced the effective number of competitors from four to three, and the remaining generator can set its own price.

IV. COMPLEX CONSTRAINT INTERACTIONS AND NODAL PRICING

In the previous case, topology had a major effect in creating the non-competitive situation outside the load pocket. The case we review now illustrates an even more complex situation, where the interplay between congestion, reactive dispatch, and voltage limits conjugate and create a rather anomalous situation. The system used to illustrate this is another modification of the 30-bus system and is shown in Figure 5. This time, it is Area 2 which is isolated by congestion. Generator #6 has decided to withdraw all but the first block of power from the market. The transmission line joining buses #4 and #12 is maximally loaded and because it is instrumental in the transfer of power from Area 1 to Area 2, it is important that most of its MVA capacity be used to transfer real power. In other words, it needs to be VAr-compensated to unity power factor on both ends, which imposes an important constraint on how generators #2, #6, and, to a lesser extent, #1, can be reactively dispatched. However,

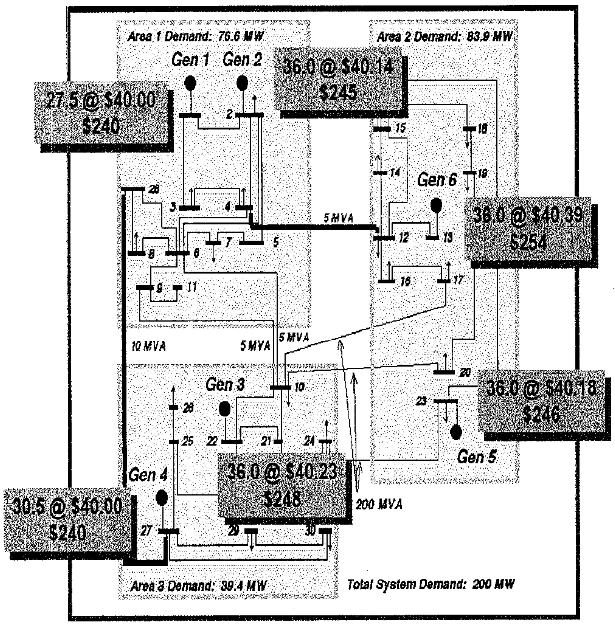

Fig. 2. Cascading market power: marginal offers

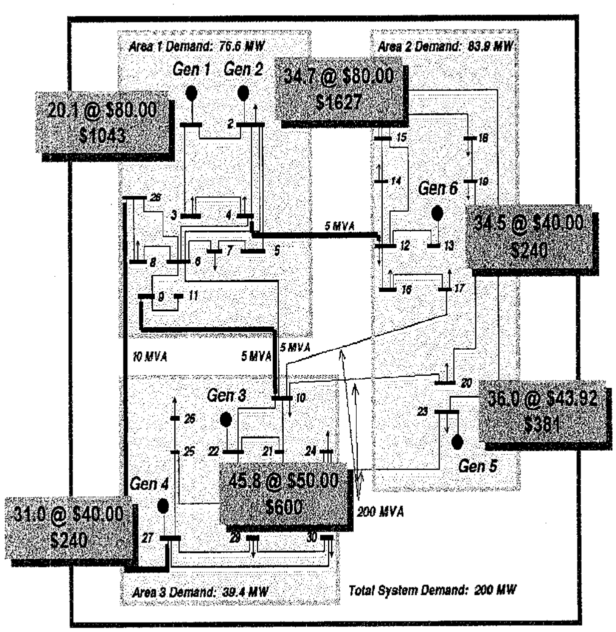

Fig. 3. Cascading market power: duopoly in Area 1

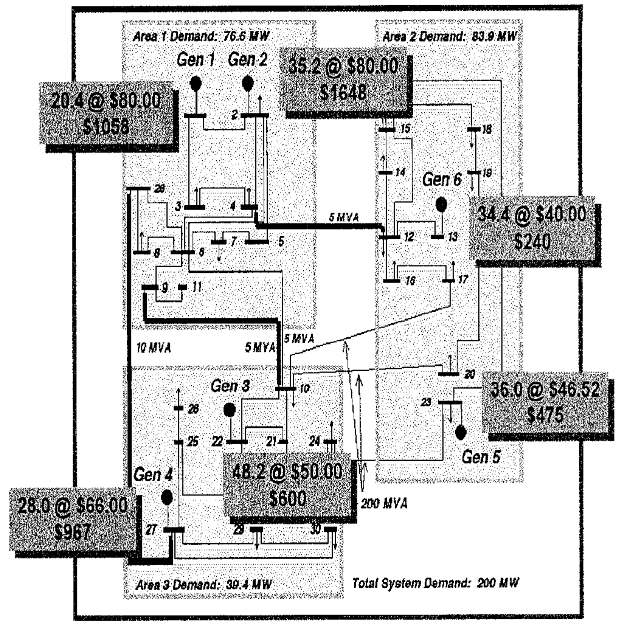

Fig. 4. Cascading market power: Generator 4 exerts its market power

the high loads on buses #16 and #17 strain the voltage profile on bus #17, and soon the reactive dispatch on generator #6 must respond to two conflicting requirements: raise VAr production to alleviate voltage problems at bus #17, or decrease it to better compensate the near end of line 4-12 and use its full transmission capability. It is not possible to achieve both simultaneously because of the path through which bus #17 is connected to bus #13; it goes through bus #12, whose voltage is directly tied to how well line 4-12 is compensated.

The implications for nodal pricing are huge. The Lagrange multipliers on the thermal MVA limit for line 4-12 are $25.78/MVAh at the left end, and $75.82/MVAh at the right end. The nodal prices for real power reach $134.56/MWh at bus #17, though the highest offer is only $50/MWh. Furthermore, the system showed a high sensitivity of dispatch to changes in load in Area 2; for an increase of 1MW at bus #17, several dozen MW must be shifted from generator #2 to generator #4, with a big increase in system cost.

V. CONCLUSIONS

The simple, yet principles-based examples shown in this paper illustrate some of the ways in which the physical operation of the grid can deviate from an idealized market exchange, with direct consequences on market power issues. Reactive power plays an important role in most of them. Also, topology can cause more problems than expected, as shown in the cascading market power case.

TABLE I
NODAL PRICES: CONSTRAINT INTERACTION EXAMPLE

bus	λ_P,\$/MWh	λ_Q, \$/MVArh
1	39.7478	0.0000
2	40.0000	0.0000
3	39.4422	0.3510
4	39.1933	0.3390
5	40.8882	-0.0100
6	41.2846	-0.2614
7	41.5289	0.0161
8	41.5118	-0.2398
9	47.3331	0.2639
10	50.3960	0.7551
11	47.3331	0.2639
12	123.2880	0.0000
13	123.2880	0.0000
14	120.0322	0.4297
15	113.3498	2.0013
16	129.4817	7.9266
17	134.5626	15.0001
18	118.2896	3.9804
19	120.5773	4.8238
20	120.7994	4.8945
21	51.5922	-0.1201
22	52.0829	0.0000
23	88.9719	0.0000
24	64.8839	0.1527
25	55.0153	-1.2057
26	56.0494	-0.5153
27	50.0000	0.0000
28	42.0391	-0.7178
29	51.4759	0.4197
30	52.5006	0.5910

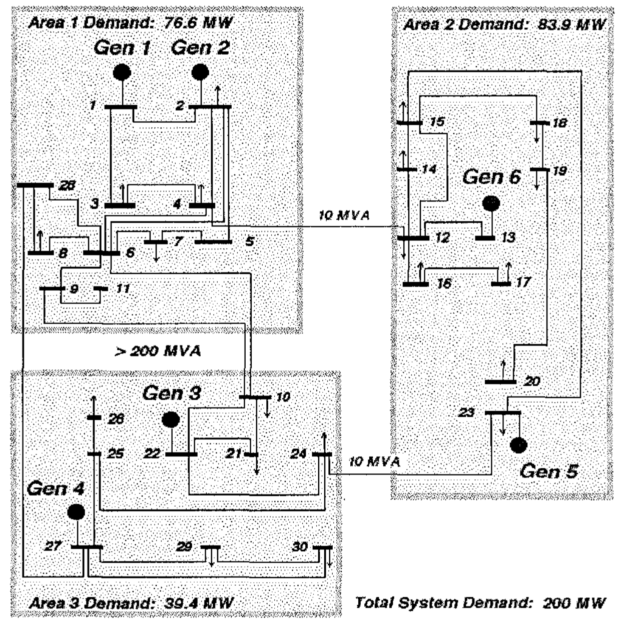

Fig. 5. Constraint interaction example

The main conclusion is that all network intricacies, including topology and reactive dispatch, must be an integral part of the market clearing mechanism, and not an afterthought.

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VI. BIOGRAPHIES

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